

## Problem 1: Haywire

Farmer John's  $N$  cows ( $4 \leq N \leq 12$ ,  $N$  even) have built a primitive system for communicating between pairs of friendly cows by building wires protected by wrappings made of hay.

Each cow has exactly 3 other friends in the barn, and the cows must arrange themselves to occupy  $N$  stalls lined up in a row. A wire of length  $L$  requires exactly  $L$  units of hay to build, so for example if the cows in stalls 4 and 7 are friends, it would take 3 units of hay to construct a wire to connect them.

Assuming every pair of friends must be connected by a separate wire, please determine the minimum possible amount of hay required to build these wires if the cows order themselves in the best possible way.

### INPUT FORMAT:

- \* Line 1: The integer  $N$ . FJ's cows are conveniently numbered  $1..N$ .
- \* Lines  $2..1+N$ : Each line contains three space-separated integers in the range  $1..N$ . Line  $i+1$  contains the numeric identifiers of the three friends of cow  $i$ . If cow  $i$  is a friend of cow  $j$ , then  $j$  will also be a friend of  $i$ .

### SAMPLE INPUT:

```
6
6 2 5
1 3 4
4 2 6
5 3 2
4 6 1
1 5 3
```

### INPUT DETAILS:

There are 6 cows. Cow 1 is friends with cows 6, 2, and 5, etc.

### OUTPUT FORMAT:

\* Line 1: The minimum total amount of hay required to connect all pairs of friendly cows.

SAMPLE OUTPUT:

17

OUTPUT DETAILS:

A best ordering of the cows is 6, 5, 1, 4, 2, 3, which requires only 17 units of hay.

## Problem 2: Goldilocks and the N Cows

You may have heard the classical story about Goldilocks and the 3 bears. Little known, however, is that Goldilocks ultimately took up farming as a profession. On her farm, she has a barn containing  $N$  cows ( $1 \leq N \leq 20,000$ ). Unfortunately, her cows are rather sensitive to temperature.

Each cow  $i$  specifies a range of temperatures  $A(i)..B(i)$  that are "just right" ( $0 \leq A(i) \leq B(i) \leq 1,000,000,000$ ). If Goldilocks sets the thermostat in the barn to a temperature  $T < A(i)$ , the cow will be too cold, and will produce  $X$  units of milk. If she sets the thermostat to a temperature  $T$  within this range ( $A(i) \leq T \leq B(i)$ ), then the cow will feel comfortable and produce  $Y$  units of milk. If she sets the thermostat to a temperature  $T > B(i)$ , the cow will feel too hot, and will produce  $Z$  units of milk. As one would expect, the value of  $Y$  is always larger than both  $X$  and  $Z$ .

Given  $X$ ,  $Y$ , and  $Z$ , as well as the preferred range of temperatures for each cow, please compute the maximum amount of milk Goldilocks can obtain if she sets the barn thermostat optimally. The values of  $X$ ,  $Y$ , and  $Z$  are integers in the range  $0..1000$ , and the thermostat can be set to any integer value.

Partial credit opportunities: Out of the 10 test cases for this problem, cases 1..4 will have  $B(i) \leq 100$  for every cow, and in cases 1..6,  $N$  is at most 1000.

### INPUT FORMAT:

\* Line 1: Four space-separated integers:  $N X Y Z$ .

\* Lines 2..1+N: Line 1+i contains two space-separated integers:  $A(i)$  and  $B(i)$ .

### SAMPLE INPUT:

```
4 7 9 6
5 8
3 4
```

13 20

7 10

INPUT DETAILS:

There are 4 cows in the barn, with temperature ranges 5..8, 3..4, 13..20, and 7..10. A cold cow produces 7 units of milk, a comfortable cow produces 9 units of milk, and a hot cow produces 6 units of milk.

OUTPUT FORMAT:

\* Line 1: The maximum amount of milk Goldilocks can obtain by an optimal temperature setting in her barn.

SAMPLE OUTPUT:

31

OUTPUT DETAILS:

If Goldilocks sets the thermostat to either 7 or 8, then she will make cows #1 and #4 happy, with cow #2 being too hot and cow #3 being too cold. This yields 31 units of total milk.

### Problem 3: Fuel Economy

Farmer John has decided to take a cross-country vacation. Not wanting his cows to feel left out, however, he has decided to rent a large truck and to bring the cows with him as well!

The truck has a large tank that can hold up to  $G$  units of fuel ( $1 \leq G \leq 1,000,000$ ). Unfortunately, it gets very poor mileage: it consumes one unit of fuel for every unit of distance traveled, and FJ has a total of  $D$  units of distance to travel along his route ( $1 \leq D \leq 1,000,000,000$ ).

Since FJ knows he will probably need to stop to refill his tank several times along his trip, he makes a list of all the  $N$  fuel stations along his route ( $1 \leq N \leq 50,000$ ). For each station  $i$ , he records its distance  $X_i$  from the start of the route ( $0 \leq X_i \leq D$ ), as well as the price  $Y_i$  per unit of fuel it sells ( $1 \leq Y_i \leq 1,000,000$ ).

Given this information, and the fact that FJ starts his journey with exactly  $B$  units of fuel ( $0 \leq B \leq D$ ), please determine the minimum amount of money FJ will need to pay for fuel in order to reach his destination. If it is impossible for him to reach the destination, please output -1. Note that the answer to this problem may not fit into a standard 32-bit integer.

#### INPUT FORMAT:

\* Line 1: Four space-separated integers:  $N$ ,  $G$ ,  $B$ , and  $D$ .

\* Lines 2..1+N: Each line contains two integers  $X_i$  and  $Y_i$  describing fuel station  $i$ .

#### SAMPLE INPUT:

```
4 10 3 17
2 40
9 15
5 7
10 12
```

#### INPUT DETAILS:

FJ is traveling along a road starting from position 0 and ending at position  $D=17$ . He starts with 3 units of fuel in a tank that can hold up to 10 units. There are 4 fuel stations; the first is at position 2 and sells fuel for a price of 40 per unit, etc.

OUTPUT FORMAT:

\* Line 1: The minimum cost FJ must pay to reach his destination, or -1 if there is no feasible way for him to reach his destination.

SAMPLE OUTPUT:

174

OUTPUT DETAILS:

FJ travels 2 units of distance and then stops to purchase 2 units of fuel (cost =  $40 \times 2$ ); this allows him to reach the station at position 5, where he fills his tank to capacity (cost =  $7 \times 10$ ). When he reaches position 10, he adds two more units of fuel (cost =  $12 \times 2$ ). The total cost is 174.

#### Problem 4: Luxury River Cruise

Farmer John is taking Bessie and the cows on a cruise! They are sailing on a network of rivers with  $N$  ports ( $1 \leq N \leq 1,000$ ) labeled  $1..N$ , and Bessie starts at port 1. Each port has exactly two rivers leading out of it which lead directly to other ports, and rivers can only be sailed one way.

At each port, the tour guides choose either the "left" river or the "right" river to sail down next, but they keep repeating the same choices over and over. More specifically, the tour guides have chosen a short sequence of  $M$  directions ( $1 \leq M \leq 500$ ), each either "left" or "right", and have repeated it  $K$  times ( $1 \leq K \leq 1,000,000,000$ ). Bessie thinks she is going in circles -- help her figure out where she ends up!

#### INPUT FORMAT:

- \* Line 1: Three space-separated integers  $N$ ,  $M$ , and  $K$ .
- \* Lines  $2..N+1$ : Line  $i+1$  has two space-separated integers, representing the number of the ports that port  $i$ 's left and right rivers lead to, respectively.
- \* Line  $N+2$ :  $M$  space-separated characters, either 'L' or 'R'. 'L' represents a choice of 'left' and 'R' represents a choice of 'right'.

#### SAMPLE INPUT:

```
4 3 3
2 4
3 1
4 2
1 3
L L R
```

#### INPUT DETAILS:

The port numbers are arranged clockwise in a circle, with 'L' being a clockwise rotation and 'R' being a counterclockwise rotation. The sequence taken is LLRLLRLLR.

OUTPUT FORMAT:

\* Line 1: A single integer giving the number of the port where Bessie's cruise ends.

SAMPLE OUTPUT:

4

OUTPUT DETAILS:

After the first iteration of the sequence of directions, Bessie is at port 2 (1 -> 2 -> 3 -> 2); after the second, she is at port 3 (2 -> 3 -> 4 -> 3), and at the end she is at port 4 (3 -> 4 -> 1 -> 4).



### Problem 5: Figure Eight

Farmer John's cows recently received a large piece of marble, which, unfortunately, has a number of imperfections. To describe these, we can represent the piece of marble by an  $N$  by  $N$  square grid ( $5 \leq N \leq 300$ ), where the character '\*' represents an imperfection and '.' represents a flawless section of the marble.

The cows want to carve a number "8" in this piece of marble (cows are quite fond of the number "8" since they have cloven hooves on each of their four feet, so they can effectively count up to 8 using their "toes"). However, the cows need your help to determine the optimally placed figure eight in the piece of marble. Here are a few properties that define a valid figure eight:

- \* A figure eight consists of two rectangles, a top and a bottom.
- \* Both the top and bottom have at least one cell in their interior.
- \* The bottom edge of the top rectangle is a (not necessarily proper) subset of the top edge of the bottom rectangle.
- \* The figure eight can only be carved from flawless regions of the piece of marble.

The aesthetic score of a figure eight is equal to the product of the areas enclosed by its two rectangles. The cows wish to maximize this score.

For example, given this piece of marble

```
.....
.....
...*****...
.*.....*.....*
.*.....*.....*
...*.....
...*.....****...
.....
..**.*.*.*...
...*.....**.*...
*..*.....*.....
.....
.....*.....
.....*.....
```

.....

the optimally placed eight is:

```

..888888888888..
..8.....8..
..8*****..8..
.*8...*.....8.*
.*8.....*...8*.
..8.*.....8..
..8*...****.8..
.88888888888888
.8**.*...*...8
.8.*...**.*...8
*8.*...*.....8
.8.....8
.8...*...*.....8
.8.....*...8
.88888888888888

```

The top rectangle has area  $6 \times 9 = 54$ , and the bottom rectangle has area  $12 \times 6 = 72$ . Thus, its aesthetic score is  $54 \times 72 = 3888$ .

INPUT FORMAT:

\* Line 1: A single integer N, indicating the side length of the marble.

\* Lines 2..N+1: Each line describes a row of the marble, and contains N characters which are each either '\*' (an imperfection) or '.' (a flawless section).

SAMPLE INPUT:

```

15
.....
.....
...*****...
.*...*.....*
.*.....*...*
....*.....

```

```

...*...****...
.....
...**.*...*...
...*...**.*...
*...*...*...
.....
.....*...*...
.....*...
.....

```

OUTPUT FORMAT:

\* Line 1: The highest aesthetic score of any figure eight which doesn't use any imperfect squares of the marble. If no figure eight is attainable, then output -1.

SAMPLE OUTPUT:

3888