Problem Set 13 Number Theory 15-295 Spring 2020

A. Number of Divisors

1 second, 256 megabytes

For a positive integer n, we use $f(n)$ to denote the number of divisors of n.

For example, $f(1) = 1,\ f(2) = 2,\ f(3) = 2,\ f(4) = 3.$

Given n, output $\sum_{i=1}^{n} f(n)$.

Input
A single integer n $(1 \leq n \leq 10^6)$.

Output
Display $\sum_{i=1}^{n} f(n)$.

input
4
output
8

B. Mertens Function

1 second, 256 megabytes

For a given positive integer n, let define $\mu(n)$ to be $\mu(n) = \begin{cases} 0 & \text{if } n \text{ has one or more repeated prime factors} \\ 1 & \text{if } n = 1 \\ (-1)^k & \text{if } n \text{ has } k \text{ distinct prime factors} \end{cases}$.

For a given positive integer n, the Mertens function is defined to be $M(n) = \sum_{i=1}^{n} \mu(n)$.

Given a given positive integer n, output the Mertens function $M(n)$.

Input
A single integer n $(1 \leq n \leq 10^7)$.

Output
Display the Mertens function $M(n)$.

input
5
output
-2

C. GCD Queries

1 second, 256 megabytes

There are n integers. Initially they are $a_1, a_2, \ldots, a_n$. There are Q modifications. The i-th modification will increase all the n integers by Δ_i. Note that Δ_i could be negative. For each modification, output the greatest common divisor of the n integers after that modification.

Input

The first line contains two integers n and Q ($1 \leq n, Q \leq 10^5$).

Each of the following n lines contains a single integer. The i-th line gives a_i.

Each of the following Q lines contains a single integer. The i-th line gives Δ_i.

It is guaranteed that all the n integers are in $[-10^{16}, 10^{16}]$ at any time.

Output

Display Q lines, where each line contains a single integer. The it-h integer is the greatest common divisor of the n integers after the i-th modification.

input

```
3 2
1
-5
7
-1
1
```

output

```
6
1
```

D. Divisible by Seven

1 second, 256 megabytes

You have number a, whose decimal representation quite luckily contains digits 1, 6, 8, 9. Rearrange the digits in its decimal representation so that the resulting number will be divisible by 7.

Number a doesn't contain any leading zeroes and contains digits 1, 6, 8, 9 (it also can contain another digits). The resulting number also mustn't contain any leading zeroes.

Input

The first line contains positive integer a in the decimal record. It is guaranteed that the record of number a contains digits: 1, 6, 8, 9. Number a doesn't contain any leading zeroes. The decimal representation of number a contains at least 4 and at most 10^6 characters.

Output

Print a number in the decimal notation without leading zeroes — the result of the permutation.

If it is impossible to rearrange the digits of the number a in the required manner, print 0.

input

```
1689
```

output

```
1869
```

input

```
18906
```

output

```
18690
```

E. Multipliers

2 seconds, 256 megabytes

Ayrat has number n, represented as it's prime factorization p_i of size m, i.e. $n = p_1 \cdot p_2 \cdot \ldots \cdot p_m$. Ayrat got secret information that that the product of all divisors of n taken modulo $10^9 + 7$ is the password to the secret data base. Now he wants to calculate this value.

Input

The first line of the input contains a single integer m ($1 \le m \le 200\,000$) — the number of primes in factorization of n.

The second line contains m primes numbers p_i ($2 \le p_i \le 200\,000$).

Output

Print one integer — the product of all divisors of n modulo $10^9 + 7$.

input

```
2
2 3
```

output

```
36
```

input

```
3
2 3 2
```

output

```
1728
```

In the first sample $n = 2 \cdot 3 = 6$. The divisors of 6 are 1, 2, 3 and 6, their product is equal to $1 \cdot 2 \cdot 3 \cdot 6 = 36$.

In the second sample $2 \cdot 3 \cdot 2 = 12$. The divisors of 12 are 1, 2, 3, 4, 6 and 12. $1 \cdot 2 \cdot 3 \cdot 4 \cdot 6 \cdot 12 = 1728$.

F. The Sum of the k-th Powers

2 seconds, 256 megabytes

There are well-known formulas: $\sum_{i=1}^{n} i = 1 + 2 + \ldots + n = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = 1^2 + 2^2 + \ldots + n^2 = \frac{n(2n+1)(n+1)}{6}$, $\sum_{i=1}^{n} i^3 = 1^3 + 2^3 + \ldots + n^3 = \left(\frac{n(n+1)}{2}\right)^2$. Also mathematicians found similar formulas for higher degrees.

Find the value of the sum $\sum_{i=1}^{n} i^k = 1^k + 2^k + \ldots + n^k$ modulo $10^9 + 7$ (so you should find the remainder after dividing the answer by the value $10^9 + 7$).

Input

The only line contains two integers n, k ($1 \le n \le 10^9, 0 \le k \le 10^6$).

Output

Print the only integer a — the remainder after dividing the value of the sum by the value $10^9 + 7$.

input
```
4 1
```
output
```
10
```

input
```
4 2
```
output
```
30
```

input
```
4 3
```
output
```
100
```

input
```
4 0
```
output
```
4
```

G. On Iteration of One Well-Known Function

1 second, 256 megabytes

Of course, many of you can calculate $\varphi(n)$ — the number of positive integers that are less than or equal to n, that are coprime with n. But what if we need to calculate $\varphi(\varphi(...\varphi(n)))$, where function φ is taken k times and n is given in the canonical decomposition into prime factors?

You are given n and k, calculate the value of $\varphi(\varphi(...\varphi(n)))$. Print the result in the canonical decomposition into prime factors.

Input

The first line contains integer m ($1 \le m \le 10^5$) — the number of distinct prime divisors in the canonical representaion of n.

Each of the next m lines contains a pair of space-separated integers p_i, a_i ($2 \le p_i \le 10^6$; $1 \le a_i \le 10^{17}$) — another prime divisor of number n and its power in the canonical representation. The sum of all a_i doesn't exceed 10^{17}. Prime divisors in the input follow in the strictly increasing order.

The last line contains integer k ($1 \le k \le 10^{18}$).

Output

In the first line, print integer w — the number of distinct prime divisors of number $\varphi(\varphi(...\varphi(n)))$, where function φ is taken k times.

Each of the next w lines must contain two space-separated integers q_i, b_i ($b_i \ge 1$) — another prime divisor and its power in the canonical representaion of the result. Numbers q_i must go in the strictly increasing order.

input
```
1
7 1
1
```
output
```
2
2 1
3 1
```

input
```
1
7 1
2
```
output
```
1
2 1
```

input
```
1
2 100000000000000000
10000000000000000
```
output
```
1
2 90000000000000000
```

You can read about canonical representation of a positive integer here:
`http://en.wikipedia.org/wiki/Fundamental_theorem_of_arithmetic`.

You can read about function $\varphi(n)$ here: `http://en.wikipedia.org/wiki/Euler's_totient_function`.