

## A. Jigsaw Puzzle

1 Point

You found a box with old games when cleaning up your attic, and among them was also a jigsaw puzzle. Unfortunately, the packaging was damaged, so a couple of puzzle pieces are scattered around the bottom of the box, and you suspect that some of the pieces may have been lost elsewhere. In fact, given the orderliness of your attic, some of the pieces in the box may even come from some entirely different puzzle! So now you have a pile of puzzle pieces lying in front of you and you are trying to assemble them into a solved puzzle.

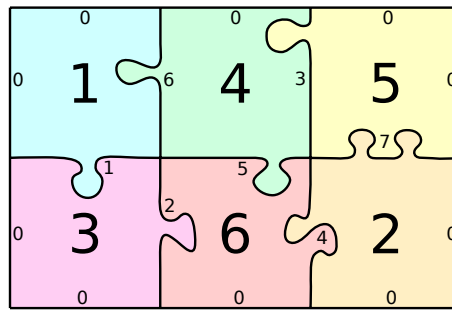


Figure J.1: Illustration of the first sample.

More formally:

- There are  $n$  square-shaped pieces, numbered from 1 to  $n$ , which need to be arranged side by side to form a single rectangle. All  $n$  pieces have to be used.
- The edges of the pieces are either straight or irregular. Straight edges must be placed on the boundary of the assembled rectangle and irregular edges must be placed on the inside.
- The irregular edges are all shaped differently, so that each edge shape occurs exactly two times, on two different puzzle pieces. Two pieces can only be placed next to each other if the shapes of the corresponding edges match.
- You may rotate the pieces, but you may not flip them over.

## Input

The input consists of:

- one line with one integer  $n$  ( $1 \leq n \leq 3 \cdot 10^5$ ), the number of pieces;
- $n$  lines, each with four integers, the  $i$ -th line gives the connections (edge shapes) of the  $i$ -th piece in counter-clockwise order.

A connection of type 0 stands for a straight edge. The other connection types are numbered with consecutive positive integers starting from 1 and each of them occurs exactly two times, on two different lines.

## Output

If the pieces cannot be assembled as described above, output `impossible`. Otherwise, output the solved puzzle in the following format:

- one line with two integers  $h, w$  ( $h, w \geq 1, h \cdot w = n$ ), the height and width of the grid;
- $h$  lines, each with  $w$  integers, the numbers of the pieces.

Any rotation of the correct solution will be accepted.

### Sample Input 1

```
6
0 0 1 6
0 7 4 0
0 0 2 1
5 3 0 6
3 7 0 0
4 5 2 0
```

### Sample Output 1

```
2 3
1 4 5
3 6 2
```

### Sample Input 2

```
4
0 0 1 2
0 0 2 3
0 0 3 4
0 0 1 4
```

### Sample Output 2

```
impossible
```

# Problem B: Battle Royale

1 Point

Battle Royale games are the current trend in video games and *Gamers Concealed Punching Circles* (GCPC) is the most popular game of them all. The game takes place in an area that, for the sake of simplicity, can be thought of as a two-dimensional plane. Movement and positioning are a substantial part of the gameplay, but getting to a desired location can be dangerous. You are confident in your ability to handle the other players, however, while you are walking to your destination, there are two hazards posed by the game itself:

- The game zone is bounded by a blue circle. Outside of this circle, there is a deadly force field that would instantly take you out of the game.
- Inside the game zone, there is a red circle where you are exposed to artillery strikes. This circle is also too risky to enter.

You want to move from one spot on the map to another, but the direct path to your destination is blocked by the red circle, so you need to find a way around it. Can you find the shortest path that avoids all hazards by never leaving the blue or entering the red circle? Touching the boundaries of the circles is fine, as long as you do not cross them.

## Input

The input consists of:

- one line with two integers  $x_c, y_c$  specifying your current location;
- one line with two integers  $x_d, y_d$  specifying your destination;
- one line with three integers  $x_b, y_b, r_b$  specifying the center and radius of the blue circle;
- one line with three integers  $x_r, y_r, r_r$  specifying the center and radius of the red circle.

All coordinates have an absolute value of at most 1 000, and  $1 \leq r_b, r_r \leq 1\,000$ . The red circle is strictly inside the blue circle. Your current location and destination are strictly inside the blue circle and strictly outside of the red circle, and the direct path between them is blocked by the red circle.

## Output

Output the length of the shortest path that does not leave the blue or enter the red circle. The output must be accurate up to a relative or absolute error (whichever is lower) of  $10^{-7}$ .

### Sample Input 1

```
0 0
10 0
0 0 1000
5 0 2
```

### Sample Output 1

```
10.8112187742
```

## C. Game Night

1 Point

It is finally Bobby's birthday, and all of his Acquaintances, Buddies and Colleagues have gathered for a board game night. They are going to play a board game which is played in up to three big teams. Bobby decided to split his guests into how well he knows them: the Acquaintances on team *A*, the Buddies on team *B*, and the Colleagues on team *C*.

While Bobby was busy explaining the rules to everyone, all his guests already took seats around his large, circular living room table. However, for the game it is crucial that all people sitting on a team are sitting next to each other. Otherwise, members of other teams could easily eavesdrop on their planning, ruining the game. So some people may need to change seats to avoid this from happening.



Bobby wants to start playing the game as soon as possible, so he wants people to switch seats as efficiently as possible. Given the current arrangement around the circular table, can you figure out the minimal number of people that must switch seats so that the teams are lined up correctly?

### Input

- The first line of the input contains the integer  $n$ , where  $1 \leq n \leq 10^5$  is the number of players (as well as seats).
- The second line contains a string of length  $n$ , consisting only of the characters in ABC. This indicates the teams of the people sitting around the table in order.

### Output

Print a single integer: the minimal number of people you have to ask to move seats to make sure the teams sit together.

#### Sample Input 1

5 ABABC	2
------------	---

#### Sample Output 1

#### Sample Input 2

12 ABCABCABCABC	6
--------------------	---

#### Sample Output 2

#### Sample Input 3

4 ACBA	0
-----------	---

#### Sample Output 3

## D. Kingpin Escape

2 Points

You are the kingpin of a large network of criminal hackers. Legend has it there has never been a richer criminal than you. Not just because you are the smartest, but also because you are the stingiest.

The police have been after you for years, but they have never been able to catch you thanks to your great set of escape routes. Whenever they want to catch you in one of your many hideouts, you quickly get away through your network of tunnels, back-alleys and speakeasies. Your routes are set up so that from every hideout you have in the city you can get to any other hideout by following only your secret passageways. Furthermore, because you are such a penny-pincher, your network is the smallest possible: from every hideout to every other hideout there is precisely *one* route through the network, no more and no fewer.



Yesterday, your mole in the police force has informed you of an unfortunate fact: the police are on to you! They have gotten wind of your secret network, and will attempt to catch you. They are planning to block some of your escape routes, and catch you in the act. They will start blocking your secret passageways one by one, until you have nowhere left to go.

Fortunately, your headquarters are absolutely safe. If you can just get there, you are always fine. Furthermore, your mole in the police force can inform you immediately as soon as the police start blocking passageways, so that they only have time to block one of them before you get notified. If you get back to your headquarters before they block any more routes, you're safe.

You want to add some passageways to the network so that whenever at most one of them is blocked, you can still get to your headquarters from any other hideout. Since the news has not changed your frugality, you want to extend your network as cheaply as possible. Can you figure out the least number of passageways you need to add, and which ones you need?

### Input

- The input starts with two integers  $2 \leq n \leq 10^5$ , the number of hideouts in the network, and  $0 \leq h < n$ , the location of your headquarters.
- Then follow  $n - 1$  lines, each with two integers  $0 \leq a, b < n$ , signifying that there is an escape route between location  $a$  and location  $b$ .

### Output

The output consists of:

- An integer  $m$ , the least number of escape routes you need to add to make the network safe again.

- Then,  $m$  lines with two integers  $0 \leq a, b < n$  each, the hideouts between which one of the escape routes has to be added.

In case there are multiple solutions, any of them will be accepted.

**Sample Input 1**

4 0	2
0 1	3 2
0 2	3 1
0 3	

**Sample Output 1**

**Sample Input 2**

6 0	2
0 1	3 5
0 2	2 4
0 3	
1 4	
1 5	

**Sample Output 2**

## E. In Case of Invasion

3 Points

After Curiosity discovered not just water on Mars, but also an aggressive, bloodthirsty bunch of aliens, the Louvain-la-Neuve municipal government decided to take precautionary measures; they built shelters in order to shelter everyone in the city in the event of an extraterrestrial attack.



Several alien-proof shelters have been erected throughout the city, where citizens can weather an alien invasion. However, due to municipal regulations and local building codes the shelters are limited in size. This makes it necessary for the government to assign every citizen a shelter to calmly direct themselves towards in the rare event of a fleet of UFOs blotting out the sun. Conditional on no shelter being assigned more people than it can fit, it is of the utmost importance that the time it takes until everyone has arrived at a shelter is minimized.

We model Louvain-la-Neuve as a network of  $n$  locations at which people live, connected by  $m$  bidirectional roads. Located at  $s$  points throughout the city are the shelters, each with a given maximum capacity. What is the minimum amount of time it takes for everyone to arrive at a shelter, when we assign people to shelters optimally?

The Louvain-la-Neuve municipal government has made sure that there is enough shelter capacity for its citizens and all shelters can be reached from any location, i.e. it is always possible to shelter everyone in some way.

### Input

- On the first line are three integers, the number of locations  $1 \leq n \leq 10^5$ , roads  $0 \leq m \leq 2 \cdot 10^5$ , and shelters  $1 \leq s \leq 10$ .
- Then follows a line with  $n$  integers  $0 \leq p_i \leq 10^9$ , indicating the the number of people living at location  $1 \leq i \leq n$ .
- Then follow  $m$  lines containing three integers  $1 \leq u, v \leq n$  and  $1 \leq w \leq 10^9$  indicating that there is a bidirectional road connecting  $u$  and  $v$  that takes  $w$  time to traverse. For any two locations there is at most one road connecting them directly, and no road connects a location to itself.
- Finally follow  $s$  lines with two integers  $1 \leq s_i \leq n$  and  $1 \leq c_i \leq 10^9$ , indicating that there is a shelter with capacity  $c_i$  at location  $s_i$ .

### Output

Print the minimum amount of time it takes to shelter everyone.

**Sample Input 1**

```
2 1 1
3 2
1 2 4
1 6
```

**Sample Output 1**

4

**Sample Input 2**

```
4 5 2
2 0 0 2
1 2 6
1 3 2
2 3 3
3 4 4
4 2 6
3 2
2 2
```

**Sample Output 2**

5

**Sample Input 3**

```
7 8 3
0 1 1 1 1 0 2
1 2 1
2 3 1
3 1 1
4 6 5
4 3 1
6 7 10
7 5 3
5 6 3
6 5
1 1
2 1
```

**Sample Output 3**

6

**Sample Input 4**

```
2 1 1
0 2
1 2 1000000000
2 2
```

**Sample Output 4**

0



## F. Entirely Unsorted

3 Points

You have recently been promoted to lead scientist at NASA, the National Association for Sorting Algorithms. Congratulations! Your primary responsibility is testing the sorting algorithms that your team produces. Fortunately, NASA has a large budget this year, and you were able to buy some state of the art integers you can use to test the sorting algorithms.

As the lead scientist, you are well aware that algorithms are tested by their behaviour on worst case inputs. So, to test sorting algorithms, you need sequences that are as unsorted as possible.

Given a sequence of numbers  $(a_1, \dots, a_n)$  we say that an element  $a_k$  is sorted if for all indices  $j$  such that  $j > k$ ,  $a_j \geq a_k$  and for all indices  $j$  such that  $j < k$ ,  $a_j \leq a_k$ . For example, in

$$(1, 3, 2, 3, 4, 6, 5, 5)$$

the sorted elements are the 1, the second occurrence of 3, and the 4. Note that a sequence is sorted if and only if all its elements are sorted. A sequence is called *entirely unsorted* if none of its elements are sorted.

Given a sequence of integers, what is the number of entirely unsorted sequences you can make by permuting its elements? Two sequences  $(b_1, \dots, b_n)$  and  $(c_1, \dots, c_n)$  are considered to be different if there is some index  $i \in \{1, \dots, n\}$  for which  $b_i \neq c_i$ . Because the number of permutations may be very large, please give it modulo  $10^9 + 9$ .

### Input

The input starts with an integer  $1 \leq n \leq 5000$ . Then follows a single line with  $n$  integers  $a_1, \dots, a_n$ , with  $0 \leq a_i \leq 10^9$  for all  $i$ .

### Output

Print a single integer: the number of entirely unsorted sequences you can make by permuting the  $a_i$ , modulo  $10^9 + 9$ .



Figure 2: Image via Flickr by Heather Paul, 'warriorwoman531', CC BY-ND 2.0.

#### Sample Input 1

```
4
0 1 2 3
```

#### Sample Output 1

```
14
```

#### Sample Input 2

```
5
1 1 2 1 1
```

#### Sample Output 2

```
1
```

## G. Heretical ... Mobius

3 Points

Rikka was walking around the school building curiously until a strange room with a door number of 404 caught her eyes.

It seemed like a computer room — there were dozens of computers lying orderly, but papers, pens, and whiteboards everywhere built up a nervous atmosphere. Suddenly, Rikka found some mysterious codes displayed on a computer which seemed to have nothing different from others — is this a message from *inner world*?

Excited Rikka started her exploration. The message was generated by a program named *for\_patterns\_in\_mobius* which outputted a string  $s$  of length  $10^9$ , containing the value of  $|\mu(x)|$  for  $x = 1, 2, \dots, 10^9$  in order.

Suddenly, Rikka heard footsteps outside. She quickly took a screenshot and left. The screenshot recorded a string  $t$  of length 200, perhaps a substring of  $s$ . Now Rikka wonders if it is really a substring of  $s$ , and if so, where it first appears in  $s$ .

Could you help her to decipher the codes?

### Input

There are 10 lines in total. Each line contains 20 characters, each of which is either “0” or “1”.  $t$  is the concatenation of them — the result of concatenating them in order.

### Output

Output a single integer in the only line. If  $t$  is a substring of  $s$ , output the first position it appears in  $s$ , that is, the minimum positive integer  $p$  such that all the digits  $|\mu(p+i)|$  for  $i = 0, 1, \dots, 199$  form the string  $t$ . Otherwise output  $-1$ .

#### Sample Input 1

```
11101110011011101010
11100100111011101110
11100110001010101110
11001110111011001110
01101110101011101000
11101110111011100110
01100010111011001110
11101100101001101110
```

#### Sample Output 1

```
1
```

10101110010011001110 11101110011011101010	
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Sample Input 2	Sample Output 2
01010101010101010101 10101010101010101010 01010101010101010101 10101010101010101010 01010101010101010101 10101010101010101010 01010101010101010101 10101010101010101010 01010101010101010101 10101010101010101010 01010101010101010101 10101010101010101010	-1

### Hint

The definition of  $\mu()$  is as follows:

For any positive integer  $x$ , let  $x = \prod_{i=1}^k p_i^{c_i}$  be the regular factorization of  $x$ , where  $p_i$  is a unique prime,  $c_i$  is a positive integer, and if  $x = 1$  then  $k = 0$ . Consequently,  $\mu(x)$  is defined as

$$\mu(x) = \begin{cases} 0 & \exists c_i > 1, \\ (-1)^k & \text{otherwise} \end{cases}$$