15-295 Spring 2019 #10 Counting and Probability

A. Archer

2 seconds, 256 megabytes

SmallR is an archer. SmallR is taking a match of archer with Zanoes. They try to shoot in the target in turns, and SmallR shoots first. The probability of shooting the target each time is $\frac{a}{b}$ for SmallR while $\frac{c}{d}$ for Zanoes. The one who shoots in the target first should be the winner.

Output the probability that SmallR will win the match.

Input

A single line contains four integers a, b, c, d $(1 \le a, b, c, d \le 1000, 0 < \frac{a}{b} < 1, 0 < \frac{c}{d} < 1)$.

Output

Print a single real number, the probability that SmallR will win the match.

The answer will be considered correct if the absolute or relative error doesn't exceed 10^{-6}.

input
1 2 1 2
output
0.666666666667

B. Wet Shark and Flowers

2 seconds, 256 megabytes

There are n sharks who grow flowers for Wet Shark. They are all sitting around the table, such that sharks i and $i+1$ are neighbours for all i from 1 to $n - 1$. Sharks n and 1 are neighbours too.

Each shark will grow some number of flowers s_i. For i-th shark value s_i is random integer equiprobably chosen in range from l_i to r_i. Wet Shark has it's favourite prime number p, and he really likes it! If for any pair of **neighbouring** sharks i and j the product $s_i \cdot s_j$ is divisible by p, then Wet Shark becomes happy and gives 1000 dollars to each of these sharks.

At the end of the day sharks sum all the money Wet Shark granted to them. Find the expectation of this value.

Input

The first line of the input contains two space-separated integers n and p ($3 \le n \le 100\,000$, $2 \le p \le 10^9$) — the number of sharks and Wet Shark's favourite prime number. It is guaranteed that p is prime.

The i-th of the following n lines contains information about i-th shark — two space-separated integers l_i and r_i ($1 \le l_i \le r_i \le 10^9$), the range of flowers shark i can produce. Remember that s_i is chosen equiprobably among all integers from l_i to r_i, inclusive.

Output

Print a single real number — the expected number of dollars that the sharks receive in total. You answer will be considered correct if its absolute or relative error does not exceed 10^{-6}.

Namely: let's assume that your answer is a, and the answer of the jury is b. The checker program will consider your answer correct, if $\frac{|a-b|}{\max(1,b)} \le 10^{-6}$.

input

```
3 2
1 2
420 421
420420 420421
```

output

```
4500.0
```

input

```
3 5
1 4
2 3
11 14
```

output

```
0.0
```

A prime number is a positive integer number that is divisible only by 1 and itself. 1 is not considered to be prime.

Consider the first sample. First shark grows some number of flowers from 1 to 2, second sharks grows from 420 to 421 flowers and third from 420420 to 420421. There are eight cases for the quantities of flowers (s_0, s_1, s_2) each shark grows:

1. $(1, 420, 420420)$: note that $s_0 \cdot s_1 = 420$, $s_1 \cdot s_2 = 176576400$, and $s_2 \cdot s_0 = 420420$. For each pair, 1000 dollars will be awarded to each shark. Therefore, each shark will be awarded 2000 dollars, for a total of 6000 dollars.
2. $(1, 420, 420421)$: now, the product $s_2 \cdot s_0$ is not divisible by 2. Therefore, sharks s_0 and s_2 will receive 1000 dollars, while shark s_1 will receive 2000. The total is 4000.
3. $(1, 421, 420420)$: total is 4000
4. $(1, 421, 420421)$: total is 0.
5. $(2, 420, 420420)$: total is 6000.
6. $(2, 420, 420421)$: total is 6000.
7. $(2, 421, 420420)$: total is 6000.
8. $(2, 421, 420421)$: total is 4000.

The expected value is $\frac{6000+4000+4000+0+6000+6000+6000+4000}{8} = 4500$.

In the second sample, no combination of quantities will garner the sharks any money.

C. Game on Tree

1 second, 256 megabytes

Momiji has got a rooted tree, consisting of n nodes. The tree nodes are numbered by integers from 1 to n. The root has number 1. Momiji decided to play a game on this tree.

The game consists of several steps. On each step, Momiji chooses one of the remaining tree nodes (let's denote it by v) and removes all the subtree nodes with the root in node v from the tree. Node v gets deleted as well. The game finishes when the tree has no nodes left. In other words, the game finishes after the step that chooses the node number 1.

Each time Momiji chooses a new node uniformly among all the remaining nodes. Your task is to find the expectation of the number of steps in the described game.

Input

The first line contains integer n $(1 \leq n \leq 10^5)$ — the number of nodes in the tree. The next $n - 1$ lines contain the tree edges. The i-th line contains integers a_i, b_i $(1 \leq a_i, b_i \leq n; a_i \neq b_i)$ — the numbers of the nodes that are connected by the i-th edge.

It is guaranteed that the given graph is a tree.

Output

Print a single real number — the expectation of the number of steps in the described game.

The answer will be considered correct if the absolute or relative error doesn't exceed 10^{-6}.

input
2 1 2
output
1.50000000000000000000

input
3 1 2 1 3
output
2.00000000000000000000

In the first sample, there are two cases. One is directly remove the root and another is remove the root after one step. Thus the expected steps are:

$$1 \times (1 / 2) + 2 \times (1 / 2) = 1.5$$

In the second sample, things get more complex. There are two cases that reduce to the first sample, and one case cleaned at once. Thus the expected steps are:

$$1 \times (1 / 3) + (1 + 1.5) \times (2 / 3) = (1 / 3) + (5 / 3) = 2$$

D. Game with String

2 seconds, 256 megabytes

Vasya and Kolya play a game with a string, using the following rules. Initially, Kolya creates a string s, consisting of small English letters, and uniformly at random chooses an integer k from a segment $[0, len(s) - 1]$. He tells Vasya this string s, and then shifts it k letters to the left, i. e. creates a new string $t = s_{k+1}s_{k+2}... s_ns_1s_2... s_k$. Vasya does not know the integer k nor the string t, but he wants to guess the integer k. To do this, he asks Kolya to tell him the first letter of the new string, and then, after he sees it, open one more letter on some position, which Vasya can choose.

Vasya understands, that he can't guarantee that he will win, but he wants to know the probability of winning, if he plays optimally. He wants you to compute this probability.

Note that Vasya wants to know the value of k uniquely, it means, that if there are at least two cyclic shifts of s that fit the information Vasya knowns, Vasya loses. Of course, at any moment of the game Vasya wants to maximize the probability of his win.

Input

The only string contains the string s of length l $(3 \le l \le 5000)$, consisting of small English letters only.

Output

Print the only number — the answer for the problem. You answer is considered correct, if its absolute or relative error does not exceed 10^{-6}.

Formally, let your answer be a, and the jury's answer be b. Your answer is considered correct if $\frac{|a-b|}{\max(1,|b|)} \le 10^{-6}$

input
technocup
output
1.000000000000000

input
tictictactac
output
0.333333333333333

input
bbaabaabbb
output
0.100000000000000

In the first example Vasya can always open the second letter after opening the first letter, and the cyclic shift is always determined uniquely.

In the second example if the first opened letter of t is "t" or "c", then Vasya can't guess the shift by opening only one other letter. On the other hand, if the first letter is "i" or "a", then he can open the fourth letter and determine the shift uniquely.

E. LRU

2 seconds, 256 megabytes

While creating high loaded systems one should pay a special attention to caching. This problem will be about one of the most popular caching algorithms called LRU (Least Recently Used).

Suppose the cache may store no more than k objects. At the beginning of the workflow the cache is empty. When some object is queried we check if it is present in the cache and move it here if it's not. If there are more than k objects in the cache after this, the least recently used one should be removed. In other words, we remove the object that has the smallest time of the last query.

Consider there are n videos being stored on the server, all of the same size. Cache can store no more than k videos and caching algorithm described above is applied. We know that any time a user enters the server he pick the video i with probability p_i. The choice of the video is independent to any events before.

The goal of this problem is to count for each of the videos the probability it will be present in the cache after 10^{100} queries.

Input

The first line of the input contains two integers n and k $(1 \le k \le n \le 20)$ — the number of videos and the size of the cache respectively. Next line contains n real numbers p_i $(0 \le p_i \le 1)$, each of them is given with no more than two digits after decimal point.

It's guaranteed that the sum of all p_i is equal to 1.

Output

Print n real numbers, the i-th of them should be equal to the probability that the i-th video will be present in the cache after 10^{100} queries. You answer will be considered correct if its absolute or relative error does not exceed 10^{-6}.

Namely: let's assume that your answer is a, and the answer of the jury is b. The checker program will consider your answer correct, if $\frac{|a-b|}{\max(1,b)} \le 10^{-6}$.

input

```
3 1
0.3 0.2 0.5
```

output

```
0.3 0.2 0.5
```

input

```
2 1
0.0 1.0
```

output

```
0.0 1.0
```

input

```
3 2
0.3 0.2 0.5
```

output

```
0.675 0.4857142857142857 0.8392857142857143
```

input

```
3 3
0.2 0.3 0.5
```

output

```
1.0 1.0 1.0
```

F. Devu and Flowers

4 seconds, 256 megabytes

Devu wants to decorate his garden with flowers. He has purchased n boxes, where the i-th box contains f_i flowers. All flowers in a single box are of the same color (hence they are indistinguishable). Also, no two boxes have flowers of the same color.

Now Devu wants to select **exactly** s flowers from the boxes to decorate his garden. Devu would like to know, in how many different ways can he select the flowers from each box? Since this number may be very large, he asks you to find the number modulo $(10^9 + 7)$.

Devu considers two ways different if there is at least one box from which different number of flowers are selected in these two ways.

Input

The first line of input contains two space-separated integers n and s $(1 \le n \le 20, 0 \le s \le 10^{14})$.

The second line contains n space-separated integers $f_1, f_2, \dots f_n$ $(0 \le f_i \le 10^{12})$.

Output

Output a single integer — the number of ways in which Devu can select the flowers modulo $(10^9 + 7)$.

input

```
2 3
1 3
```

output

```
2
```

input

```
2 4
2 2
```

output

```
1
```

input

```
3 5
1 3 2
```

output

```
3
```

Sample 1. There are two ways of selecting 3 flowers: $\{1, 2\}$ and $\{0, 3\}$.

Sample 2. There is only one way of selecting 4 flowers: $\{2, 2\}$.

Sample 3. There are three ways of selecting 5 flowers: $\{1, 2, 2\}$, $\{0, 3, 2\}$, and $\{1, 3, 1\}$.