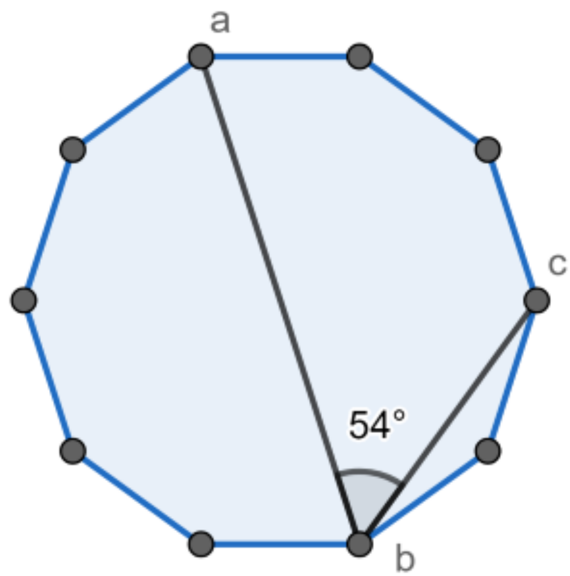
15-295 Spring 2019 #9 Geometry

A. Polygon for the Angle

2 seconds, 256 megabytes

You are given an angle ang.

The Jury asks You to find such **regular** n-gon (regular polygon with n vertices) that it has three vertices a, b and c (they can be non-consecutive) with $\angle abc = \text{ang}$ or report that there is no such n-gon.



If there are several answers, print the **minimal** one. It is guarantied that if answer exists then it doesn't exceed 998244353.

Input

The first line contains single integer T ($1 \le T \le 180$) — the number of queries.

Each of the next T lines contains one integer ang ($1 \le \text{ang} < 180$) — the angle measured **in degrees**.

Output

For each query print single integer n ($3 \le n \le 998244353$) — minimal possible number of vertices in the regular n-gon or -1 if there is no such n.

input
4 54 50 2 178
output
10 18 90 180

The answer for the first query is on the picture above.

The answer for the second query is reached on a regular 18-gon. For example, $\angle v_2 v_1 v_6 = 50^\circ$.

The example angle for the third query is $\angle v_{11} v_{10} v_{12} = 2^\circ$.

In the fourth query, minimal possible n is 180 (not 90).

B. Area of Two Circles' Intersection

2 seconds, 256 megabytes

You are given two circles. Find the area of their intersection.

Input

The first line contains three integers x_1, y_1, r_1 ($-10^9 \le x_1, y_1 \le 10^9$, $1 \le r_1 \le 10^9$) — the position of the center and the radius of the first circle.

The second line contains three integers x_2, y_2, r_2 ($-10^9 \le x_2, y_2 \le 10^9$, $1 \le r_2 \le 10^9$) — the position of the center and the radius of the second circle.

Output

Print the area of the intersection of the circles. The answer will be considered correct if the absolute or relative error doesn't exceed 10^{-6}.

input

```
0 0 4
6 0 4
```

output

```
7.25298806364175601379
```

input

```
0 0 5
11 0 5
```

output

```
0.00000000000000000000
```

C. Blowing Candles

As Jacques-Édouard really likes birthday cakes, he celebrates his birthday every hour, instead of every year. His friends ordered him a round cake from a famous pastry shop, and placed candles on its top surface. The number of candles equals the age of Jacques-Édouard in hours. As a result, there is a huge amount of candles burning on the top of the cake. Jacques-Édouard wants to blow all the candles out in one single breath.

You can think of the flames of the candles as being points in the same plane, all within a disk of radius R (in nanometers) centered at the origin. On that same plane, the air blown by Jacques-Édouard follows a trajectory that can be described by a straight strip of width W, which comprises the area between two parallel lines at distance W, the lines themselves being included in that area. What is the minimum width W such that Jacques-Édouard can blow all the candles out if he chooses the best orientation to blow?

Input

The first line consists of the integers N and R, separated with a space, where N is Jacques-Édouard's age in hours. Then N lines follow, each of them consisting of the two integer coordinates x_i and y_i of the ith candle in nanometers, separated with a space.

Limits

- $3 \leqslant N \leqslant 2 \cdot 10^5$;
- $10 \leqslant R \leqslant 2 \cdot 10^8$;
- for $1 \leqslant i \leqslant N$, $x_i^2 + y_i^2 \leqslant R^2$;
- all points have distinct coordinates.

Output

Print the value W as a floating point number. An additive or multiplicative error of 10^{-5} is tolerated: if y is the answer, any number either within $[y - 10^{-5}; y + 10^{-5}]$ or within $[(1 - 10^{-5})y; (1 + 10^{-5})y]$ is accepted.

Sample Input

```
3 10
0 0
10 0
0 10
```

Sample Output

```
7.0710678118654755
```

D. The Fair Nut and Rectangles

2 seconds, 256 megabytes

The Fair Nut got stacked in planar world. He should solve this task to get out.

You are given n rectangles with vertexes in $(0, 0)$, $(x_i, 0)$, (x_i, y_i), $(0, y_i)$. For each rectangle, you are also given a number a_i. Choose some of them that the area of union minus sum of a_i of the chosen ones is maximum.

It is guaranteed that there are no nested rectangles.

Nut has no idea how to find the answer, so he asked for your help.

Input

The first line contains one integer n $(1 \le n \le 10^6)$ — the number of rectangles.

Each of the next n lines contains three integers x_i, y_i and a_i $(1 \le x_i, y_i \le 10^9, 0 \le a_i \le x_i \cdot y_i)$.

It is guaranteed that there are no nested rectangles.

Output

In a single line print the answer to the problem — the maximum value which you can achieve.

input

```
3
4 4 8
1 5 0
5 2 10
```

output

```
9
```

input

```
4
6 2 4
1 6 2
2 4 3
5 3 8
```

output

```
10
```

In the first example, the right answer can be achieved by choosing the first and the second rectangles.

In the second example, the right answer can also be achieved by choosing the first and the second rectangles.

Problem E. Gears

Input file:	`stdin`
Output file:	`stdout`
Time limit:	2 seconds
Memory limit:	256 megabytes

There are two polygons on the plane, A and B. Polygon A rotates around point P, and polygon B rotates around point Q. Each polygon rotates with the constant rotational speed in the clockwise direction around its point, the rotational speed values of the polygons' rotation are equal.

Your task is to determine if there will be a collision between polygons. A collision is a situation when the polygons have at least one common point.

It is guaranteed that at the moment 0 the polygons A and B do not intersect and no polygon is fully contained inside another one.

Note that:

- the polygons are not necessarily convex;
- points P and Q can be located on the border of or outside their polygons.

Input

The first line contains space-separated coordinates of point P.

The second line contains a single integer n ($3 \le n \le 1000$) — the number of vertices of polygon A.

Each of the next n lines contains two space-separated integers — the coordinates of the corresponding vertex of polygon A.

The next line is empty.

Then follow space-separated coordinates of point Q.

The next line contains a single integer m ($3 \le m \le 1000$) — the number of vertices of polygon B. Next m lines contain the coordinates of the vertices of the polygon B.

The vertices of both polygons are listed in the counterclockwise order. Coordinates of all points are integers, their absolute values don't exceed 10^4.

Output

Print "`YES`", if the collision takes place and "`NO`" otherwise (don't print the quotes).

F. New Year Snowflake

2 seconds, 256 megabytes

As Gerald ..., in other words, on a New Year Eve Constantine prepared an unusual present for the Beautiful Lady. The present is the magic New Year snowflake that can make any dream come true.

The New Year snowflake consists of tiny ice crystals, which can be approximately regarded as points on the plane. The beauty of the New Year snowflake is that it has a center of symmetry. This is a point such that for each crystal of the snowflake exists another crystal, symmetrical to it relative to that point. One of the crystals can be placed directly in the center of symmetry.

While Constantine was choosing a snowflake among millions of other snowflakes, no less symmetrical and no less magical, then endured a difficult path through the drifts to the house of his mistress, while he was waiting with bated breath for a few long moments before the Beautiful Lady opens the door, some of the snowflake crystals melted and naturally disappeared. Constantine is sure that there were no more than k of such crystals, because he handled the snowflake very carefully. Now he is ready to demonstrate to the Beautiful Lady all the power of nanotechnology and restore the symmetry of snowflakes.

You are given the coordinates of the surviving snowflake crystals, given in nanometers. Your task is to identify all possible positions of the original center of symmetry.

Input

The first line contains two integers n and k ($1 \le n \le 200\,000$, $0 \le k \le 10$) — the number of the surviving snowflake crystals and the maximum number of melted crystals, correspondingly. Next n lines contain the coordinates of the crystals that are left in the following form: "x_i y_i". The coordinates are integers and do not exceed $5 \cdot 10^8$ in absolute value. All given points are different.

Output

The first line contains an integer c — the number of possible symmetry centers. Next c lines should contain the centers' descriptions. Each symmetry center is described by a couple of coordinates "x y", separated by a space. Print the coordinates with absolute error not exceeding 10^{-6}. You are allowed to print the symmetry centers in any order. All printed points should be different. If there exist an infinite number of possible symmetry centers, print the single number "-1".

input

```
4 0
0 0
0 1
1 0
1 1
```

output

```
1
0.5 0.5
```

input

```
4 2
0 0
0 1
1 0
1 1
```

output

```
5
0.0 0.5
0.5 0.0
0.5 0.5
0.5 1.0
1.0 0.5
```

input

```
4 4
0 0
0 1
1 0
1 1
```

output

```
-1
```