15-295 Fall 2018 #12 Geometry

A. View Angle

2 seconds, 256 megabytes

Flatland has recently introduced a new type of an eye check for the driver's licence. The check goes like that: there is a plane with mannequins standing on it. You should tell the value of the minimum angle with the vertex at the origin of coordinates and with all mannequins standing inside or on the boarder of this angle.

As you spend lots of time "glued to the screen", your vision is impaired. So you have to write a program that will pass the check for you.

Input

The first line contains a single integer n $(1 \le n \le 10^5)$ — the number of mannequins.

Next n lines contain two space-separated integers each: x_i, y_i $(|x_i|, |y_i| \le 1000)$ — the coordinates of the i-th mannequin. It is guaranteed that the origin of the coordinates has no mannequin. It is guaranteed that no two mannequins are located in the same point on the plane.

Output

Print a single real number — the value of the sought angle in degrees. The answer will be considered valid if the relative or absolute error doesn't exceed 10^{-6}.

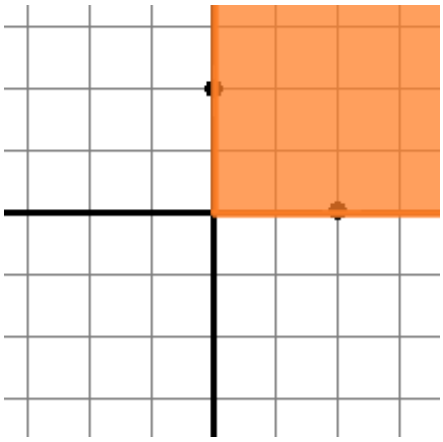
input
2 2 0 0 2
output
90.0000000000

input
3 2 0 0 2 -2 2
output
135.0000000000

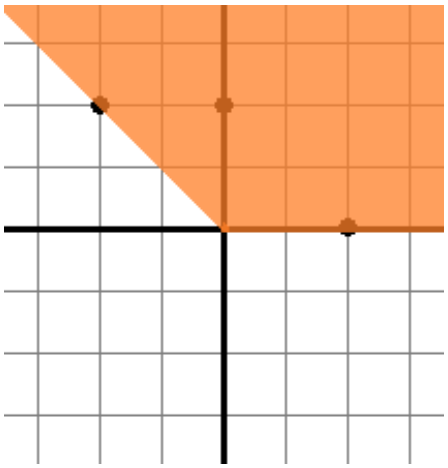
input
4 2 0 0 2 -2 0 0 -2
output
270.0000000000

input
2 2 1 1 2
output
36.8698976458

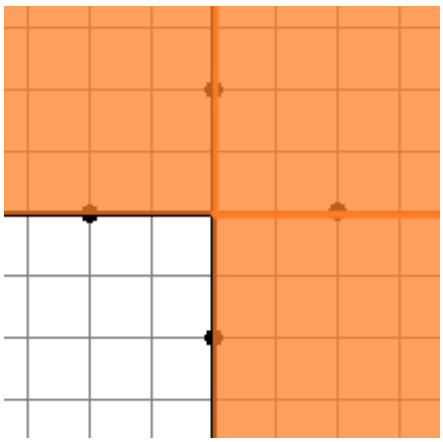
Solution for the first sample test is shown below:



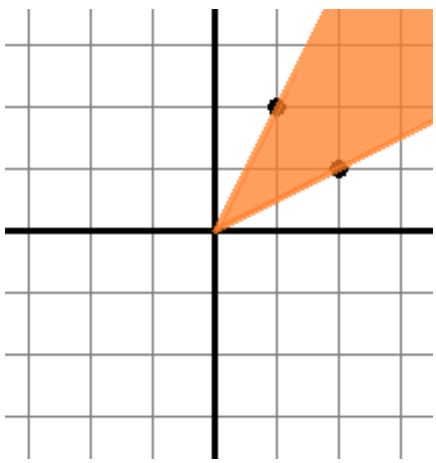
Solution for the second sample test is shown below:



Solution for the third sample test is shown below:



Solution for the fourth sample test is shown below:

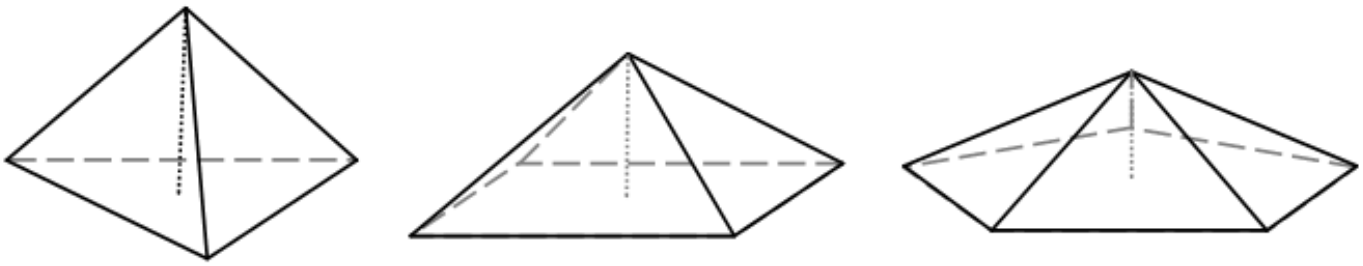


B. Pyramids

0.5 seconds, 64 megabytes

IT City administration has no rest because of the fame of the Pyramids in Egypt. There is a project of construction of pyramid complex near the city in the place called Emerald Walley. The distinction of the complex is that its pyramids will be not only quadrangular as in Egypt but also triangular and pentagonal. Of course the amount of the city budget funds for the construction depends on the pyramids' volume. Your task is to calculate the volume of the pilot project consisting of three pyramids — one triangular, one quadrangular and one pentagonal.

The first pyramid has equilateral triangle as its base, and all 6 edges of the pyramid have equal length. The second pyramid has a square as its base and all 8 edges of the pyramid have equal length. The third pyramid has a regular pentagon as its base and all 10 edges of the pyramid have equal length.



Input

The only line of the input contains three integers l_3, l_4, l_5 ($1 \le l_3, l_4, l_5 \le 1000$) — the edge lengths of triangular, quadrangular and pentagonal pyramids correspondingly.

Output

Output one number — the total volume of the pyramids. Absolute or relative error should not be greater than 10^{-9}.

input
2 5 3
output
38.546168065709

C. Polygons

2 seconds, 256 megabytes

You've got another geometrical task. You are given two non-degenerate polygons A and B as vertex coordinates. Polygon A is strictly convex. Polygon B is an arbitrary polygon without any self-intersections and self-touches. The vertices of both polygons are given in the clockwise order. For each polygon no three consecutively following vertices are located on the same straight line.

Your task is to check whether polygon B is positioned strictly inside polygon A. It means that any point of polygon B should be strictly inside polygon A. "Strictly" means that the vertex of polygon B cannot lie on the side of the polygon A.

Input

The first line contains the only integer n ($3 \le n \le 10^5$) — the number of vertices of polygon A. Then n lines contain pairs of integers x_i, y_i ($|x_i|, |y_i| \le 10^9$) — coordinates of the i-th vertex of polygon A. The vertices are given in the clockwise order.

The next line contains a single integer m ($3 \le m \le 2 \cdot 10^4$) — the number of vertices of polygon B. Then following m lines contain pairs of integers x_j, y_j ($|x_j|, |y_j| \le 10^9$) — the coordinates of the j-th vertex of polygon B. The vertices are given in the clockwise order.

The coordinates of the polygon's vertices are separated by a single space. It is guaranteed that polygons A and B are non-degenerate, that polygon A is strictly convex, that polygon B has no self-intersections and self-touches and also for each polygon no three consecutively following vertices are located on the same straight line.

Output

Print on the only line the answer to the problem — if polygon B is strictly inside polygon A, print "`YES`", otherwise print "`NO`" (without the quotes).

input
6 -2 1 0 3 3 3 4 1 3 -2 2 -2 4 0 1 2 2 3 1 1 0
output
YES

input

```
5
1 2
4 2
3 -3
-2 -2
-2 1
4
0 1
1 2
4 1
2 -1
```

output

```
NO
```

input

```
5
-1 2
2 3
4 1
3 -2
0 -3
5
1 0
1 1
3 1
5 -1
2 -1
```

output

```
NO
```

D. Large Triangle

3 seconds, 256 megabytes

> There is a strange peculiarity: if you connect the cities of Rostov, Taganrog and Shakhty, peculiarly, you get a triangle
>
> «Unbelievable But True»

Students from many different parts of Russia and abroad come to Summer Informatics School. You marked the hometowns of the SIS participants on a map.

Now you decided to prepare an interesting infographic based on this map. The first thing you chose to do is to find three cities on this map, such that they form a triangle with area S.

Input

The first line of input contains two integers n and S ($3 \le n \le 2000$, $1 \le S \le 2 \cdot 10^{18}$) — the number of cities on the map and the area of the triangle to be found.

The next n lines contain descriptions of the cities, one per line. Each city is described by its integer coordinates x_i, y_i ($-10^9 \le x_i, y_i \le 10^9$).

It is guaranteed that all cities are located at distinct points. It is also guaranteed that no three cities lie on the same line.

Output

If the solution doesn't exist — print «No».

Otherwise, print «Yes», followed by three pairs of coordinates (x, y) — the locations of the three cities, which form the triangle of area S.

input
3 7 0 0 3 0 0 4
output
No

input
4 3 0 0 2 0 1 2 1 3
output
Yes 0 0 1 3 2 0

Problem E
Connect the Dots
Time limit: 2 seconds

A famous logical problem is that of connecting 9 dots on a paper by drawing 4 line segments with a pencil, while never lifting the pencil from the paper. While this is easy enough (although it requires some thinking outside of the box), Simone has recently been building a game called "Connect the Dots" around a generalisation of the concept.

In Connect the Dots, you are presented with a 4×4 regular grid of dots. Each dot is given a unique number between 1 and 16. The task is then to connect the dots in order by their numbers, starting with dot 1 and ending with dot 16. The dots should be connected using *as few line segments as possible*, starting at dot 1, with the end of each segment being the start point of the next. The segments are allowed to intersect and overlap one another. Additionally, it is allowed to pass through other points while trying to connect the current point. This means, for example, that visiting the first four points in the order $1, 4, 2, 3, 2, 4, \ldots$ is acceptable. Formally, the sequence $1, 2, \ldots, 15, 16$ must be a subsequence of the sequence of dots visited.

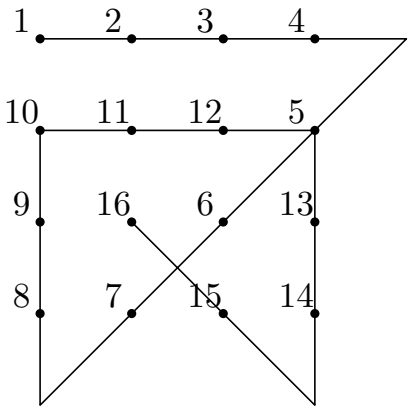


Figure C.1: A solution to the first sample.

Simone asked you to try the puzzle out, while betting you a balloon that it would be too hard. Prove her wrong by writing a program that solves the puzzle for you!

Input

The input consists of:

- 4 lines, each with 4 integers, the numbers of the dots in the grid. The jth number on the ith line is the number of the jth dot in the ith row of the grid of dots.

The 16 numbers in the input are all between 1 and 16 (inclusive) and pairwise distinct.

Output

Output the minimum number of line segments needed to connect all the dots in order.

Sample Input 1

```
1 2 3 4
10 11 12 5
9 16 6 13
8 7 15 14
```

Sample Output 1

```
6
```

Sample Input 2

```
1 2 3 4
8 9 10 11
7 15 16 12
6 14 13 5
```

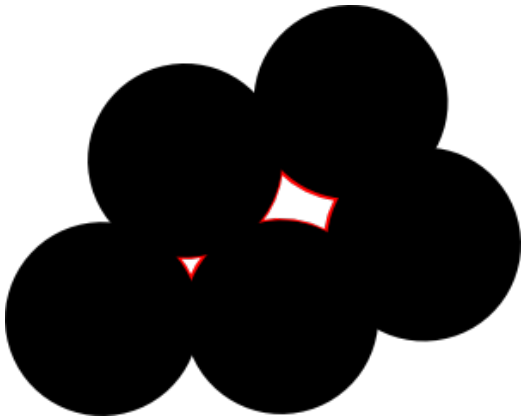
Sample Output 2

```
7
```

F. The Last Hole!

2 seconds, 256 megabytes

Luyi has n circles on the plane. The i-th circle is centered at (x_i, y_i). At the time zero circles start to grow simultaneously. In other words, the radius of each circle at time t $(t > 0)$ is equal to t. The circles are drawn as black discs on an infinite white plane. So at each moment the plane consists of several black and white regions. Note that the circles may overlap while growing.



We define a hole as a closed, connected white region. For instance, the figure contains two holes shown by red border. During growing some holes may be created and it is easy to see that each created hole will disappear eventually. Luyi asks you to find moment of time such that the last hole disappears. In other words, you should find the first moment such that no hole can be seen after that.

Input

The first line of the input contains integer n $(1 \le n \le 100)$. Each of the next n lines contains two integers x_i and y_i $(-10^4 \le x_i, y_i \le 10^4)$, indicating the location of i-th circle.

It's guaranteed that no two circles are centered at the same point.

Output

Print the moment where the last hole disappears. If there exists no moment in which we can find holes print `-1`.

The answer will be considered correct if the absolute or relative error does not exceed 10^{-4}.

input
3 0 0 1 1 2 2
output
-1

input
4 0 0 0 2 2 2 2 0
output
1.414214

input
4 0 1 0 -1 -2 0 4 0
output
2.125000