# Problem A Candy division

Input file:	standard input
Output file:	standard output
Time limit:	2 seconds
Memory limit:	64 megabytes

Your older sister has three kids. Whenever you go to visit her, you bring along a bag of candies for the kids. There are n candies in the bag. You want to give all the candies to the kids, but you also want to teach them a little math along the way. Therefore, you gave them not just the bag of candies but also one simple rule: each of the kids must take an integer fraction of candies in the bag. In other words, the amount of candies each kid takes must be a divisor of n.

Formally, in order to divide all the candies the kids have to find three *positive* integers  $a_1, a_2, a_3$  such that  $n = a_1 + a_2 + a_3$  and each  $a_i$  divides n.

#### Input

The first line of the input contains a single integer t – the number of test cases to follow. Each of the following t lines of the input contains the integer n. You may assume that  $1 \le t \le 100$  and  $1 \le n \le 10^{18}$ .

## Output

Output t lines. The k-th line will solve the k-th test case and will contain three integers  $a_i$  as specified above. If there are multiple solutions you may select an arbitrary one. If there are no solutions, output the word 'IMPOSSIBLE' instead (quotes only for clarity).

#### Example

standard input	standard output
3	IMPOSSIBLE
1	1 1 1
3	4 6 2
12	

# Problem B. Triangle in a Triangle

Input file:	standard input
Output file:	standard output
Time limit:	4 seconds
Memory limit:	64 megabytes

This year's public viewing of SWERC will take place on the famous Triangular Plaza, which is delimited by three perfectly straight roads. Due to weather concerns you have been hired to design a rain cover for the event. This cover should of course be triangular (as is tradition for the Triangular Plaza) and cover as much of the Plaza as possible. The only other restriction is that the rain cover's corners must be fixed on 3 of the lampposts that are standing along the roads. Can you figure out the maximal area of the Plaza you can cover?

### Input

The first three lines of input contain pairs of integers, the x- and y-coordinates of the 3 corners A, B, C of the Plaza. It is guaranteed that  $0 \le x, y \le 10^6$ . The next line contains a single integer  $n \ge 1$ , the number of lampposts on the line segment  $\overline{AB}$ . The following line contains n integers  $d_i$  in ascending order, where  $d_i$  is the distance between the corner A and the *i*-th lamppost. The next two lines are of the same format but for the line segment  $\overline{BC}$  and the corner B. The last two lines are of the same format but for the line segment  $\overline{CA}$  and the corner C. It is guaranteed that the lampposts are on the segment between two corners. A lamppost may however be in a corner. In total there will be less than  $10^6$  lampposts. (In case you are worried about precision: it is also guaranteed that all triangles of lampposts that have an area of at least 70% of the solution will have angles that are all > 20 degrees).

# Output

Print a single floating point number, the maximal area of the Plaza that can be covered. The answer is considered correct if it is within a  $10^{-6}$  absolute or relative error margin.

## Example

standard input	standard output
0 0	12.000000
6 0	
6 7	
2	
1 5	
2	
2 6	
1	
5	

# Problem C Chess

Input file:	standard input
Output file:	standard output
Time limit:	2 seconds
Memory limit:	256 megabytes

A legend says that, as a reward for inventing chess, the creator asked his ruler for 1 grain of rice on the first square, 2 on the second, 4 on the third, and on each of the following twice the amount of the preceding square. The ruler at first laughed it off as a meager prize for such a remarkable invention, but soon (around the 3rd rank, where he needed more than millions of grains) realized his mistake.

Paul invented a new version of chess, that uses N squares and assigned each square a value the same way. However, for convenience, he used all the numbers modulo some positive integer M.

You have recovered a sorted sequence of these numbers  $\{A_i\}$ , and you want to find which M was used.

#### Input

First line contains a single number  $1 \le N \le 2 \cdot 10^5$ , the number of squares on the chessboard. The next line contains N sorted numbers  $0 \le A_i \le 10^{18}$ , representing the values written on squares. It is guaranteed that a solution exists.

# Output

Output a single line with a single integer M, the modulus that was used for the original calculation. If there are multiple solutions, print the smallest one.

## Examples

standard input	standard output
5	13
1 2 3 4 8	
2	3
1 2	
8	7
1 1 1 2 2 2 4 4	
24	49999
1 2 4 8 16 32 64 128 256 512 1024	
2048 4096 8192 12149 15537 16384	
24298 31074 32768 38775 44387 47193	
48596	

#### Note

In the first test case, the numbers written on squares, in order, are 1, 2, 4, 8 and 3  $(2 \cdot 8 \mod 13)$ .

In the second test case, the smallest possible solution is 3, but other numbers, like 7 or 11, would also work.

In the third case, note that the numbers can repeat and it is possible for N to be larger than M.

Fourth case sees a well-known prime in action.

# Problem D. Effective network

Input file:	standard input
Output file:	standard output
Time limit:	4 seconds
Memory limit:	256 megabytes

Quite recently, in a very close galaxy cluster, there was a bunch of planets. As the galaxy expands to a vast space, the only effective way of traveling between them is to use the network of teleporters. Due to technological problems, one can only travel between certain pairs of teleporters, using so-called links. This means that in order to travel from planet A to planet B, one may need to first teleport from A to some intermediate planet before reaching B. The distance between planets A and B is the minimum number of links required to travel between them. It is guaranteed that one can travel between each pair of planets using a finite number of links, but this number may be quite large for some pairs.

You are an employee of Teleport GmbH and were tasked to set up terms and conditions for the GA Travel Card such that the customers are happy. In order to do this, you need to select a non-empty subset of planets, called promoted planets, and designate all the links between them (and only those) to be included in the GA. We will call these free links. Thanks to customer survey you know that there are three conditions you need to fulfill:

- There should be at least two promoted planets.
- It should be possible to travel between any two promoted planets A and B using only free links
- The distance between any two promoted planets A and B, using only free links, cannot be larger than R K, where R is the number of promoted planets, and K is a fixed constant representing the demands of the customers.

Determine whether it is possible to find a set of promoted planets, and if so, return one set that satisfies the conditions. If there are multiple solutions, return any of them.

#### Input

First line contains integers  $1 \le N \le 5000$ ,  $N - 1 \le M \le \min(\frac{N(N-1)}{2}, 30000)$ ,  $1 \le K \le N$  – the number of planets, the number of links, and the height of the demands, respectively.

Next M lines contain description of links. Each link is represented by two integers,  $1 \leq u_i, v_i \leq N$ , meaning that there is a link between planets  $u_i$  and  $v_i$ . It is guaranteed that no pair occurs more than once, and also that  $u_i \neq v_i$ .

It is guaranteed that one can travel between each pair of planets using a finite number of links.

## Output

If there is a solution, output two lines: the first containing a single integer R – the number of promoted planets. On the second line, print R space separated integers, specifying the indices of promoted planets.

If there is no set of planets that would satisfy the conditions, output a single line containing the integer 0.

# Examples

standard input	standard output
7 9 3	4
1 3	1 3 4 6
1 4	
1 5	
2 5	
3 4	
6 3	
6 1	
4 6	
1 7	
5 4 3	0
1 3	
1 2	
3 4	
3 5	
561	3
1 3	1 3 5
1 2	
3 4	
3 5	
2 3	
54	

## Note

In the first sample case, we select a set of four planets that are all pairwise connected. As R = 4 and K = 3, the conditions are satisfied.

In the second test case, there is no subset of planets satisfying the conditions.

In the third case note that the pair of planets 1 and 5 would not be a solution, as one cannot travel between them without interchange at a non-promoted planet.

# Problem E Compass

Input file:	standard input
Output file:	standard output
Time limit:	2 seconds
Memory limit:	64 megabytes

The university of Algoland is located in a single huge building. It is great. It is the best building in the world. It only uses the best angles: the right angle! But the building is so huge that it is also very confusing to new students, because it is very easy to get lost.

The rector of Algoland's university, a former professor in physics, had a great idea to prevent students from getting lost in the future: He bought an incredibly strong magnet with the intention of placing it somewhere inside the building and using it as an emergency meeting point. On the first day of the semester, every student gets a free compass. With that compass the student can always tell the direction towards the magnet (the magnet is so strong that it completely dominates the earth's magnetic field). If a student gets lost, she can follow the following simple procedure to get to the emergency meeting point at the magnet's location:

- Move straight into the direction towards the magnet until you either reach the magnet, or bump into a wall.
- If your path is blocked by a wall, follow the direction alongside the wall that brings you closer towards the magnet, until you either reach the end of the wall or your path becomes orthogonal to the compass direction.
- If the wall is perfectly orthogonal to the compass direction, or you end up in a corner, you are stuck. Scream as loud as you can!

In the corner case where you want to walk parallel to a wall at the exact coordinate of a wall, you are not stopped by the wall. We assume here that you are infinitesimally small (which is true in proportion to the size of the building).

The rector now wants your help to place the magnet inside the building in such a way that every student can reach it (following the procedure above) no matter where inside the building the student gets lost. Actually, you just have to decide if this is possible or not.

### Input

The first line contains an integer N, the number of points,  $4 \le N \le 10^3$ .

Each of the following N lines contains two coordinates x and y  $(0 \le x \le 10^3 \text{ and } 0 \le y \le 10^3)$ . Each line describes a corner point of the single wall that delimits the inside from the outside of the building. The points are given in clockwise order and all the angles are guaranteed to be 90 degrees.

# Output

Print a single line containing a single word: either SAFETY if it is possible to place the magnet such that one is always able to find it, or DANGER otherwise.

# Examples

standard input	standard output
12	SAFETY
1 0	
1.3	
2 3	
2.0	
3 2	
4 2	
4 1	
5 1	
5 3	
6 3	
6 0	
12	DANGER
04	
5 4	
5.0	
2 0	
2 0	
2 0	
3 1	
4 1	
4 3	
0 1	
14	SAFETY
6 4	
6 0	
3 0	
3 1	
2 1	
2 0	
1 0	
1 2	
4 2	
4 1	
5 1	
5 3	
1 2	
1 4	

# Note



# Problem F Affine

Input file:	standard input
Output file:	standard output
Time limit:	2 seconds
Memory limit:	64 megabytes

We call a function f from  $\mathbb{R}^m$  to  $\mathbb{R}^2$  affine if it maps straight lines to straight lines. Formally, f is affine if and only if  $f(\alpha \cdot \vec{x} + (1 - \alpha) \cdot \vec{y}) = \alpha \cdot f(\vec{x}) + (1 - \alpha) \cdot f(\vec{y})$  for all  $\alpha \in \mathbb{R}$  and  $\vec{x}, \vec{y} \in \mathbb{R}^m$ . (Note that this is a weaker condition than linearity:  $f(x_1, x_2) = 3 \cdot x_1 + 2 \cdot x_2 + 1$  is affine but not linear.)

You are given a polygon P in the plane. Determine the smallest integer  $m \ge 0$  such that there is an affine function f that maps the *m*-hypercube  $[0,1]^m$  to P, or determine that no such m exists.

#### Input

The first line of the input contains the integer N ( $1 \le N \le 100'000$ ), the number of given boundary points of the polygon. The *i*-th of the next N lines contains two integers  $X_i$  and  $Y_i$  ( $-10^8 \le X_i, Y_i \le 10^8$ ), the coordinates of the *i*-th given boundary point of the polygon.

The counterclockwise boundary of the polygon P is obtained by connecting the given points by line segments.  $(X_i, Y_i)$  is connected to  $(X_{i+1}, Y_{i+1})$  for  $1 \le i < N$ , and  $(X_N, Y_N)$  is connected to  $(X_1, Y_1)$ . All given points are distinct and the boundary of the polygon P neither intersects nor touches itself.

## Output

Print INFINITY, if there is no m such that P is an affine image of the m-hypercube. Otherwise, print a single non-negative integer, the smallest m such that P is an affine image of the m-hypercube.

standard input	standard output
4	2
0 0	
1 0	
1 1	
0 1	
6	3
0 0	
2 0	
3 1	
3 3	
1 3	
02	
2	1
-10000000 29611633	
10000000 29611633	

### Examples